Dynamic Modeling Of Self-phoretic Magnetic Janus Microrobot

Special session

Control of bio-hybrid & bio-inspired microrobotic systems

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MARSS
International Conference on Manipulation, Automation and Robotics at Small Scales

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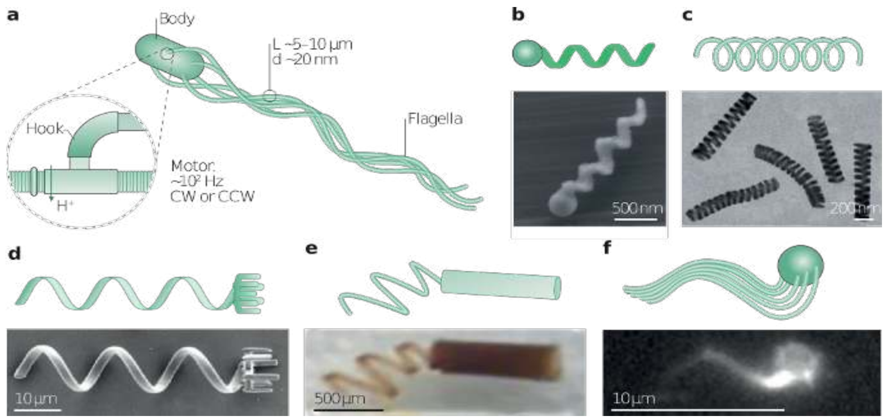
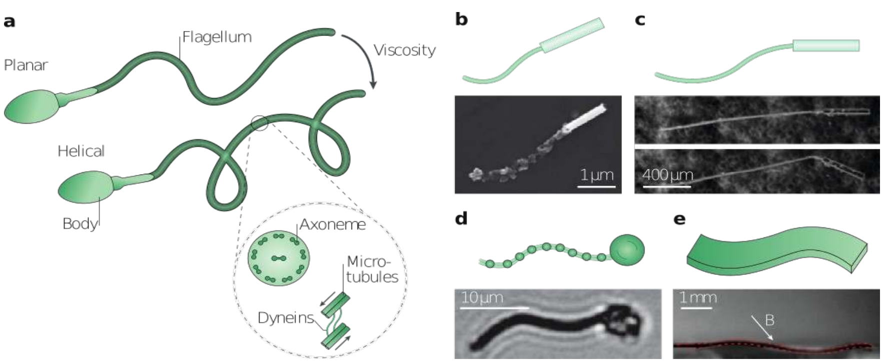
Locomotion at the microscale

- **Moving in liquids at the microscale ≠ from swimming at the macroscale**
  [E.M. Purcell, 1977]

- **Bioinspired propulsion**
  - Helical microswimmer
    - swim by rotating propellers
      (eg. bacteria flagella : spirochaetes)
  - Flexible microstructures
    - swim by moving a flexible flagella
      (e.g. sperm cells and many unicellular eukaryotes)
  - *Quid of microorganism without moving part ?*
    → (self-)phoretic motion

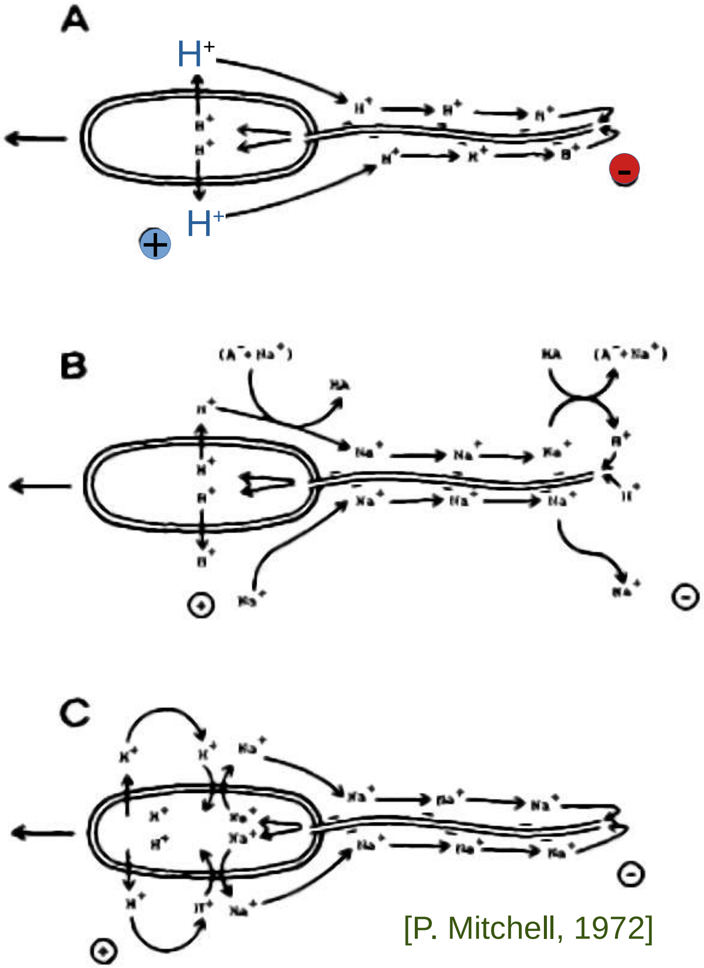




[S. Palagi and P. Fischer, 2018]

❑ Self-phoretic motion

- « Microorganisms might propel themselves through nonmechanical means without using any moving parts [P. Mitchell, 1956, 1972]
- Bacteria pump ions asymmetrically
  → an electrical circuit is formed
- Electrokinetic mechanism → (here) **self-electrophoresis**



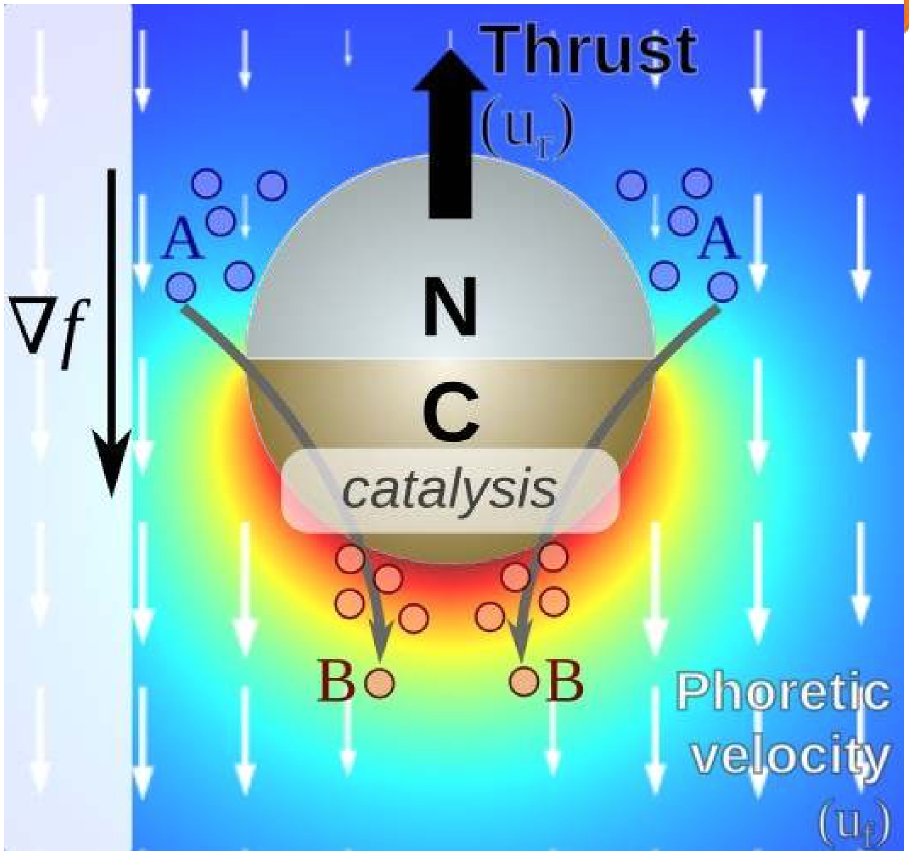
[P. Mitchell, 1972]

Modeling of Self-Phoretic Motion

- Phoretic motion

$$u_f = \mu_p (\boldsymbol{I} - \boldsymbol{nn}) \cdot \nabla f$$

  - μ_p: local *phoretic mobility*
  - ∇f: « phoretic gradient »
    - $\nabla f \equiv \nabla c$: diffusiophoresis (chemical gradient)
    - $\nabla f \equiv \nabla \Phi = E$: electrophoresis (electric field)
    - also thermophoresis...



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Modeling

Modeling of Self-Phoretic Motion

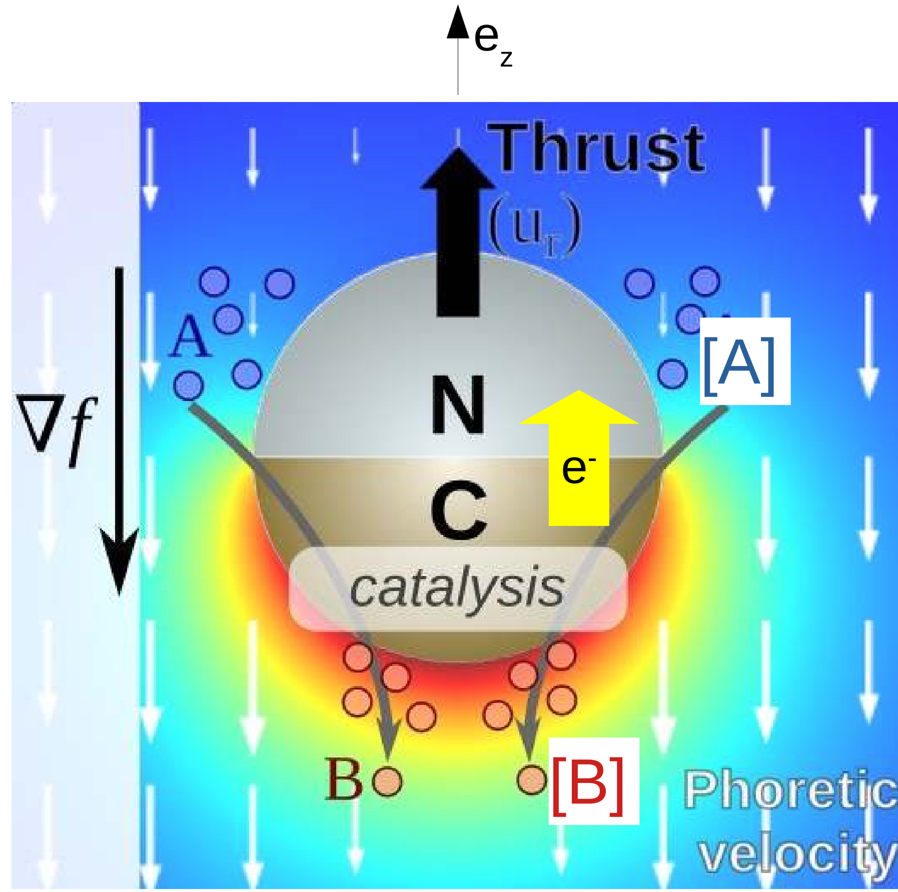
- Phoretic motion

$$u_f = \mu_p (\boldsymbol{I} - \boldsymbol{nn}) \cdot \nabla f$$

- Catalytic reaction :
  - Diffusiophoresis : (fuel) A→ B

    $$\nabla c = \left\{ \frac{d[A]}{dz} ; \frac{d[B]}{dz} \right\}$$

  - Electrophoresis : $A_{ox} + H^+ + e^- \rightarrow B_{red}$
  - **Catalyst materials**:
    - Pt, Pd, Au, Ni, Ag, Ru, etc.
    - enzymes: catalase, urease, glucose oxidase, etc.
    - *the particle must be asymmetric!* → at least two parts (e.g. N/C)
  - **Fuel**: H_2O_2, aqueous solution,strong acids, hydrazine, glucose, urea, etc.



Example : **fuel** = hydrogen peroxide, **catalyst**=Ru/Au

- *Anode :* $3H_2O_2 \rightarrow 6H^+ + 3O_2 + 6e^-$
- *Cathode :*
  $H_2O_2 + 2H^+ + 2e^- \rightarrow 2H_2O$
  $4H^+ + O_2 + 4e^- \rightarrow 2H_2O$

❑ Modeling of Self-Phoretic Motion

- Phoretic motion

$$u_f = \mu_p (\boldsymbol{I} - \boldsymbol{nn}) \cdot \nabla f$$

- Swimming velocity :

$$v_p \cdot e_z = - \iint_S d\boldsymbol{r}\, \boldsymbol{n} \cdot \sigma_z \cdot u_f$$

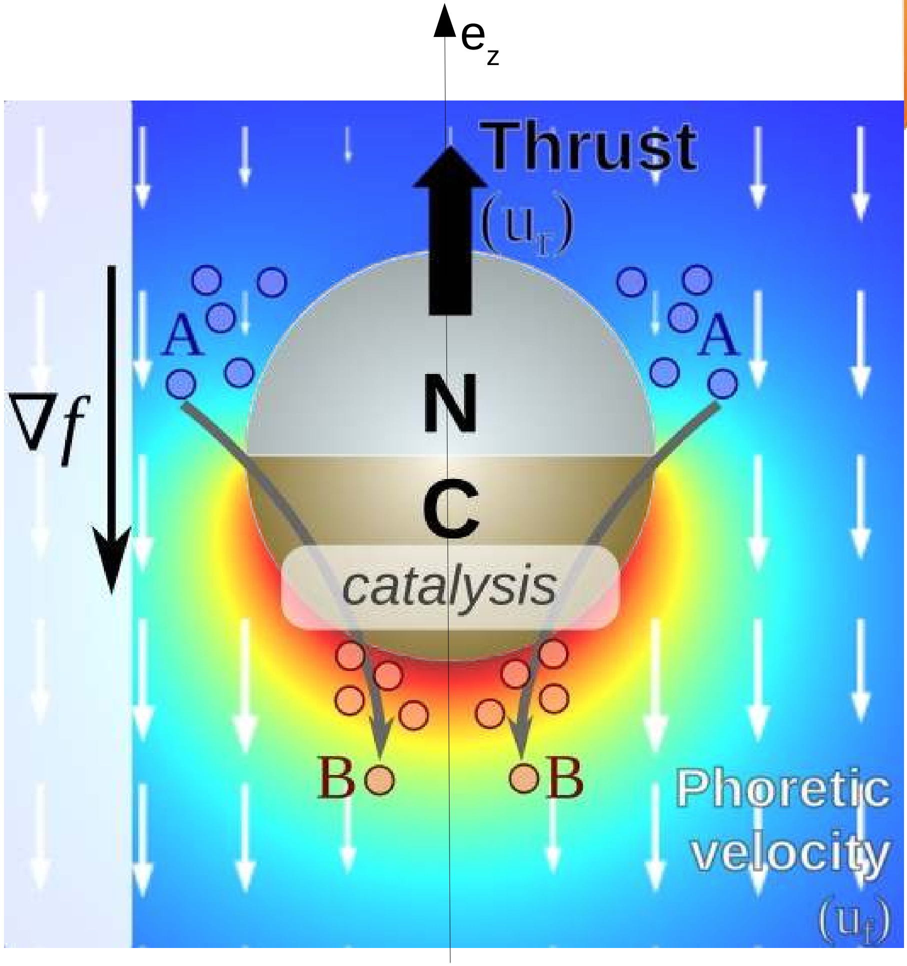
  - σ_z : hydrodynamic stress tensor at S
  - For spherical particle :

$$v_p = \frac{-1}{4\pi r^2} \iint_S d\boldsymbol{r}\, u_f(r)$$

  - e.g. $v_p = u_0 e_z$

$$u_0 \propto -\mu_p \frac{a(r)}{D}$$

    - $D = k_B T / k_d$ diffusion coefficient (with k_d the drag coeff.)
    - a(r): surface activity of the catalytic reaction



❑ Modeling of Self-Phoretic Motion

- Phoretic motion

$$u_f = \mu_p (\boldsymbol{I} - \boldsymbol{n}\boldsymbol{n}) \cdot \nabla f$$

- Swimming velocity :

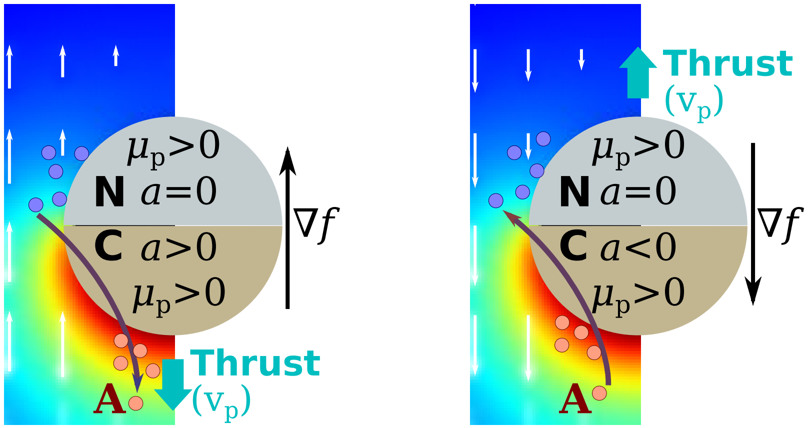
$$u_0 \propto -\mu_p \frac{a(r)}{D}$$

- Moving in viscous liquids

$$\begin{pmatrix} \boldsymbol{f}_d \\ \boldsymbol{t}_d \end{pmatrix} = -\begin{pmatrix} \boldsymbol{K} & \boldsymbol{C} \\ \boldsymbol{C}^t & \boldsymbol{\Omega} \end{pmatrix} \begin{pmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{pmatrix}$$

  - Example: for a sphere of radius r
    - $K = d_f \cdot \boldsymbol{I}$, $d_f = 6\pi\eta r$
    - $\boldsymbol{\Omega} = d_t \cdot \boldsymbol{I}$, $d_t = 8\pi\eta r^3$
    - η : viscosity

- **without other forces,** at equilibrium: $f_d + f_p = 0$

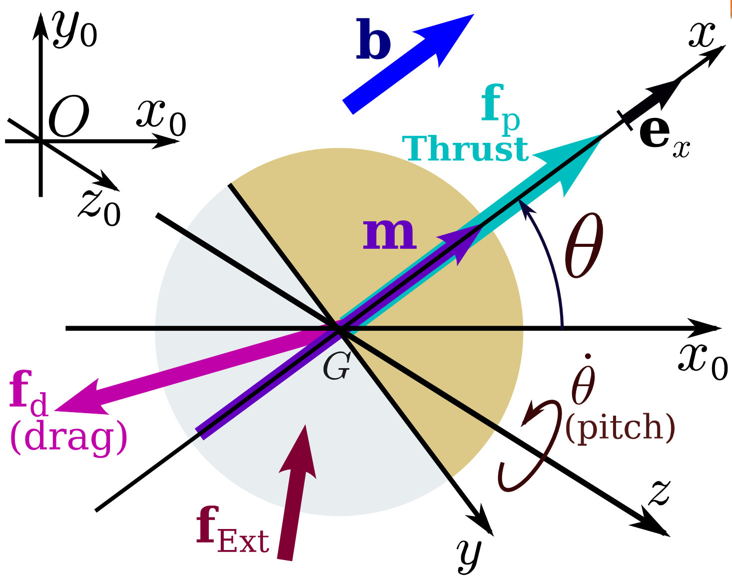


Modeling

Dynamic

- Self-phoretic propulsion: {$\mathbf{f}_p$;$\mathbf{t}_p$}
- Hydrodynamic: {$\mathbf{f}_d$;$\mathbf{t}_p$}
- Disturbances: {$\mathbf{f}_{ext}$;$\mathbf{t}_{ext}$}
  - e.g. radom fluctuation → Brownian motion
- Actuation: {$\mathbf{f}_a$;$\mathbf{t}_a$}
  - i.e. enabling external control
  - e.g. magnetic actuation
- Newton's law:

$$\begin{cases} m\dot{v} = \boldsymbol{f}_p + \boldsymbol{f}_d + \boldsymbol{f}_a + \boldsymbol{f}_{ext} \\ J\dot{\omega} = \boldsymbol{t}_p + \boldsymbol{t}_d + \boldsymbol{t}_a + \boldsymbol{t}_{ext} \end{cases}$$



❑ Dynamic

- Without external controls
  → Active Brownian Particles

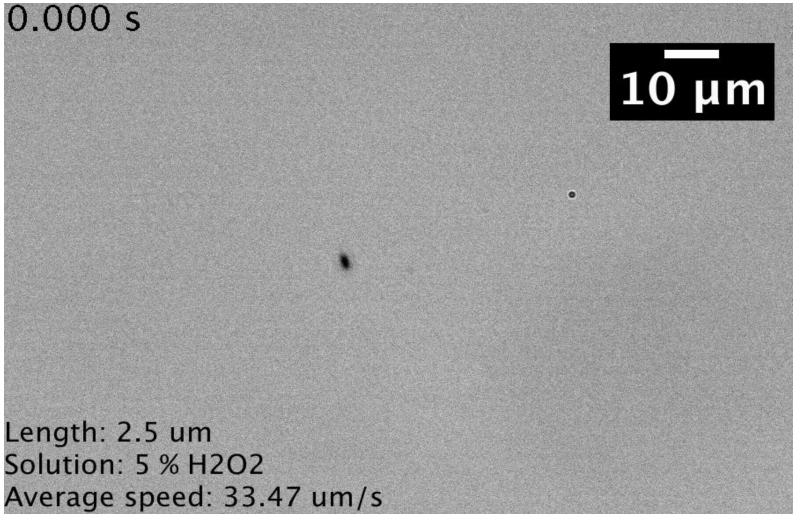
$$\begin{cases} \boldsymbol{f}_{ext} = {}^{t}\xi \\ \boldsymbol{t}_{ext} = {}^{r}\xi \end{cases}$$

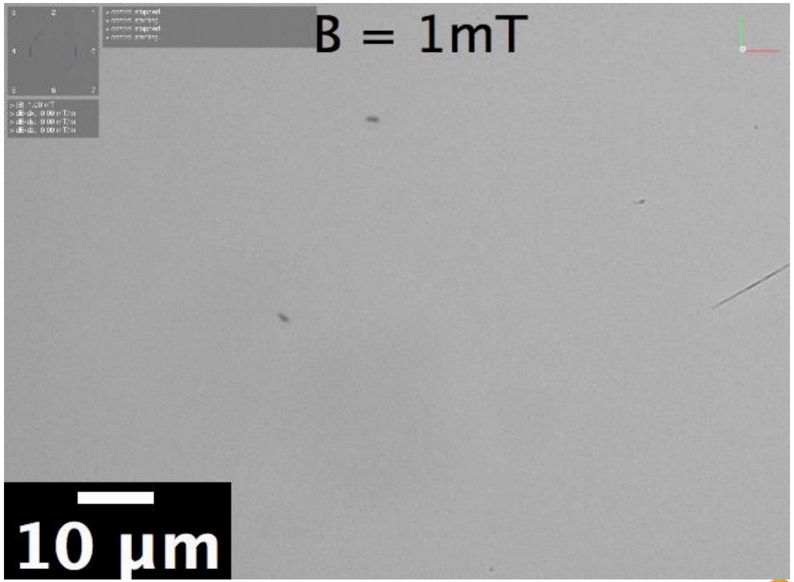
  - stochastic force and torque:

$$\begin{cases} \langle {}^{t}\xi_i(t_1), {}^{t}\xi_j(t_2) \rangle = 2 k_B T\, d_t\, \delta_{ij}(t_1 - t_2) \\ \langle {}^{r}\xi_i(t_1), {}^{r}\xi_j(t_2) \rangle = 2 k_B T\, d_r\, \delta_{ij}(t_1 - t_2) \end{cases}$$

  - … but chemotaxis is possible…
  - mainly for random sensing

- Simple solution: embed magnetic material
  - magnetic actuation
    (e.g. magnetic guidance)

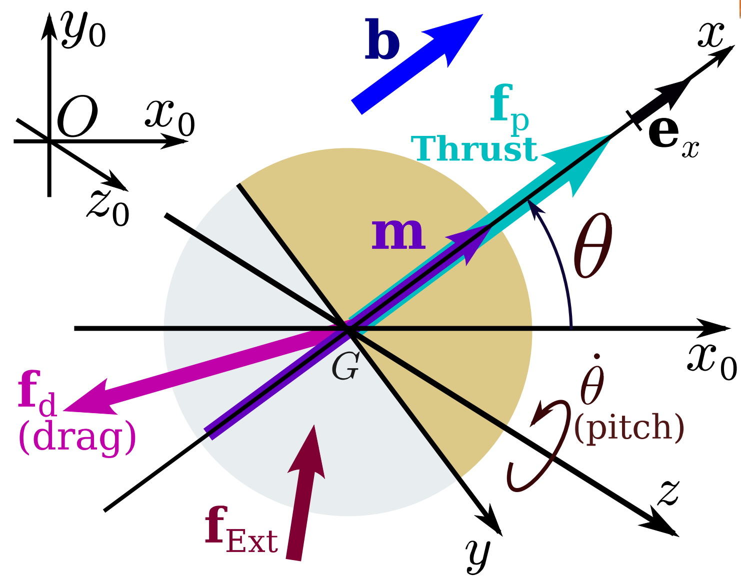




❑ Magnetic Janus microRobot (MJR)

- Spherical shape with 2 hemispheres:
  - C: catalytic part
  - N: non catalytic
  - +magnetic material

$$\begin{cases} \boldsymbol{f}_{\boldsymbol{m}} = V_m \cdot (\boldsymbol{m}\nabla)\boldsymbol{b} \\ \boldsymbol{t}_{\boldsymbol{m}} = V_m (\boldsymbol{m} \times \boldsymbol{b}) \end{cases}$$



- Simplification: 2D problem
  - Position+Orientation: (x,y,θ) (assumed measurable)
  - 2D motion: v=(vx,vy,0) and ω=(0,0,ωz)

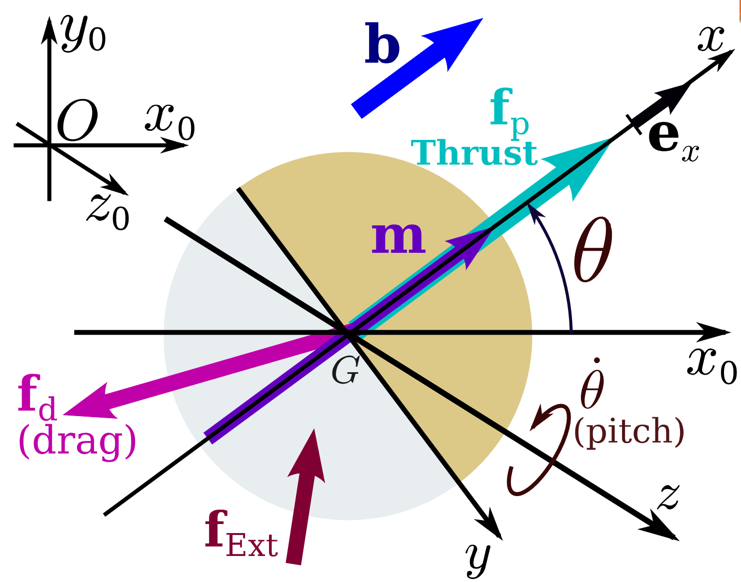
Case study (MJR)

❑ 2D state space representation

$$(S_x)\begin{cases}\dot{x}=v_x\\ \dot{v}_x=-\alpha_x v_x+\alpha_x\cos(\theta)u_0+\xi_x\end{cases}$$

$$(S_x)\begin{cases}\dot{x}=v_x\\ \dot{v}_x=-\alpha_x v_x+\alpha_x\cos(\theta)u_0+\xi_x\end{cases}$$

$$(S_\theta)\begin{cases}\dot{\theta}=\omega\\ \dot{\omega}=-\alpha_\theta\omega+\beta\left(\sin(\theta)b_x+\cos(\theta)b_y\right)+\xi_x\end{cases}$$



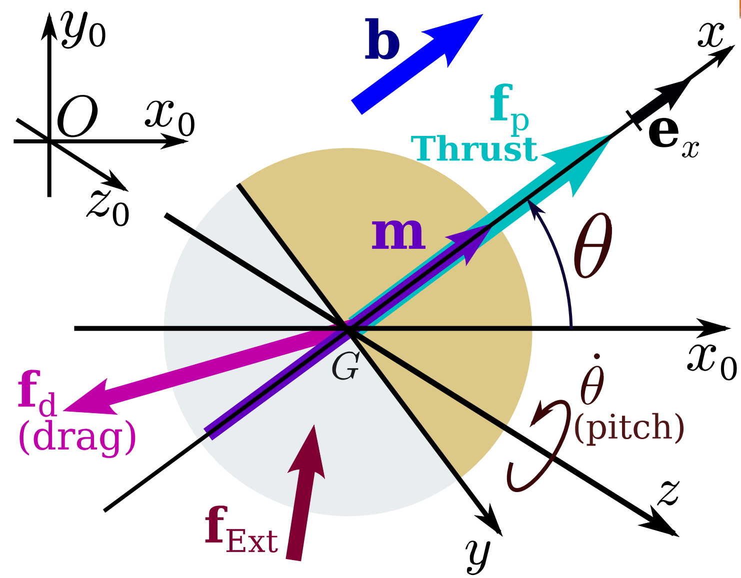
- States: $\mathbf{x}=(x,v_x,y,v_y,\theta,\omega_z)$
- Measures: $\mathbf{y}=(x,y,\theta)$
- Inputs: $\mathbf{u}=(u_0,b_x,b_y)$
  - *self-phoretic motion:* $u_p=u_0\cdot e_x$
  - *magnetic field:* $\mathbf{b}=(b_x,b_y)$

❑ Pseudo-linear system

$$\begin{cases}\dot{\boldsymbol{x}}=\boldsymbol{A}\,\boldsymbol{x}+\boldsymbol{B}(\boldsymbol{x})\,\boldsymbol{u}+\boldsymbol{W}\\ \dot{\boldsymbol{y}}=\boldsymbol{C}\,\boldsymbol{x}+\boldsymbol{v}\end{cases}$$

Discussions

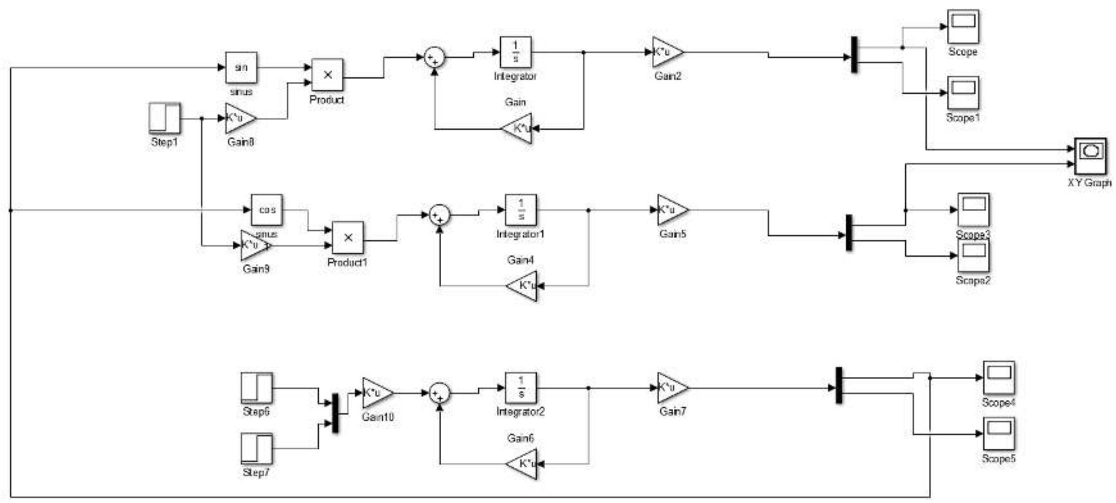
- self-phoretic motion : $v_p = u_0\, e_z$, $u_0 \propto -\mu_p \frac{a(r)}{D}$
  input or *state*?
  - precise control of u_0 difficult
  - how to adjust the surface activity a(r)?
    - *change in fuel concentration*
    - *light can enable/disable catalytic reaction*
  - But… the pair {A,B(x)} is not controllable without u_0!



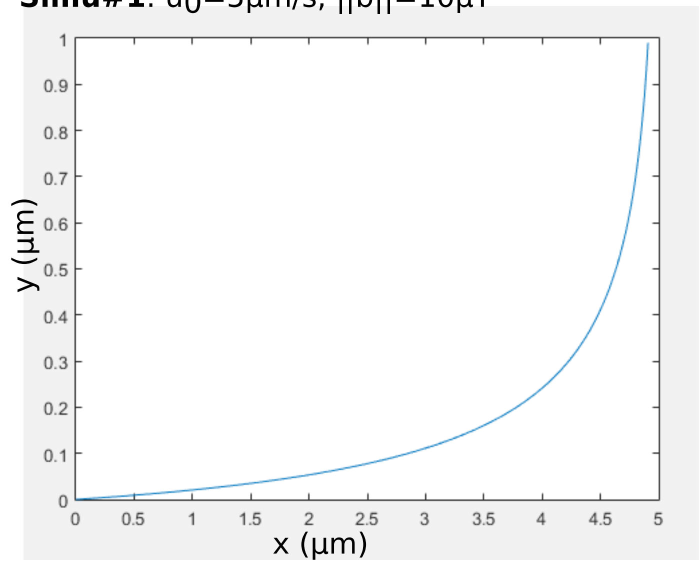
- Inertia: negligible at the microscale?
  - Deal with the transient behavior
  - Allows estimation of the velocities

Case study (MJR)

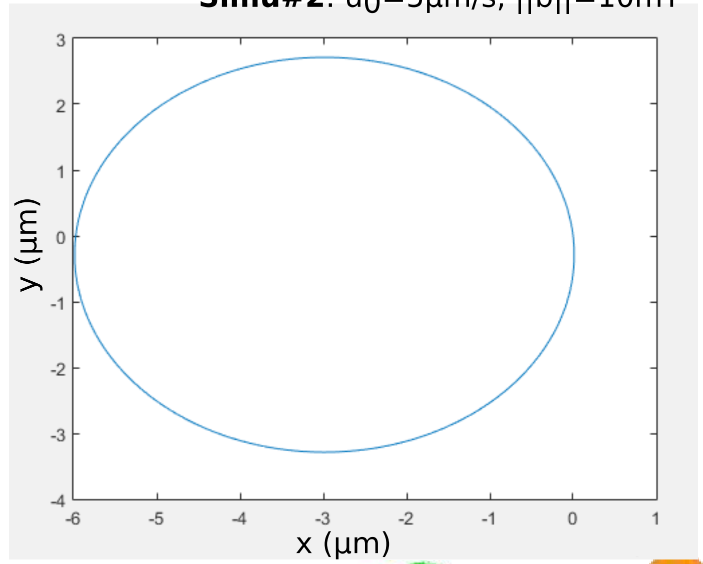
❑ Matlab/Simulink simulation



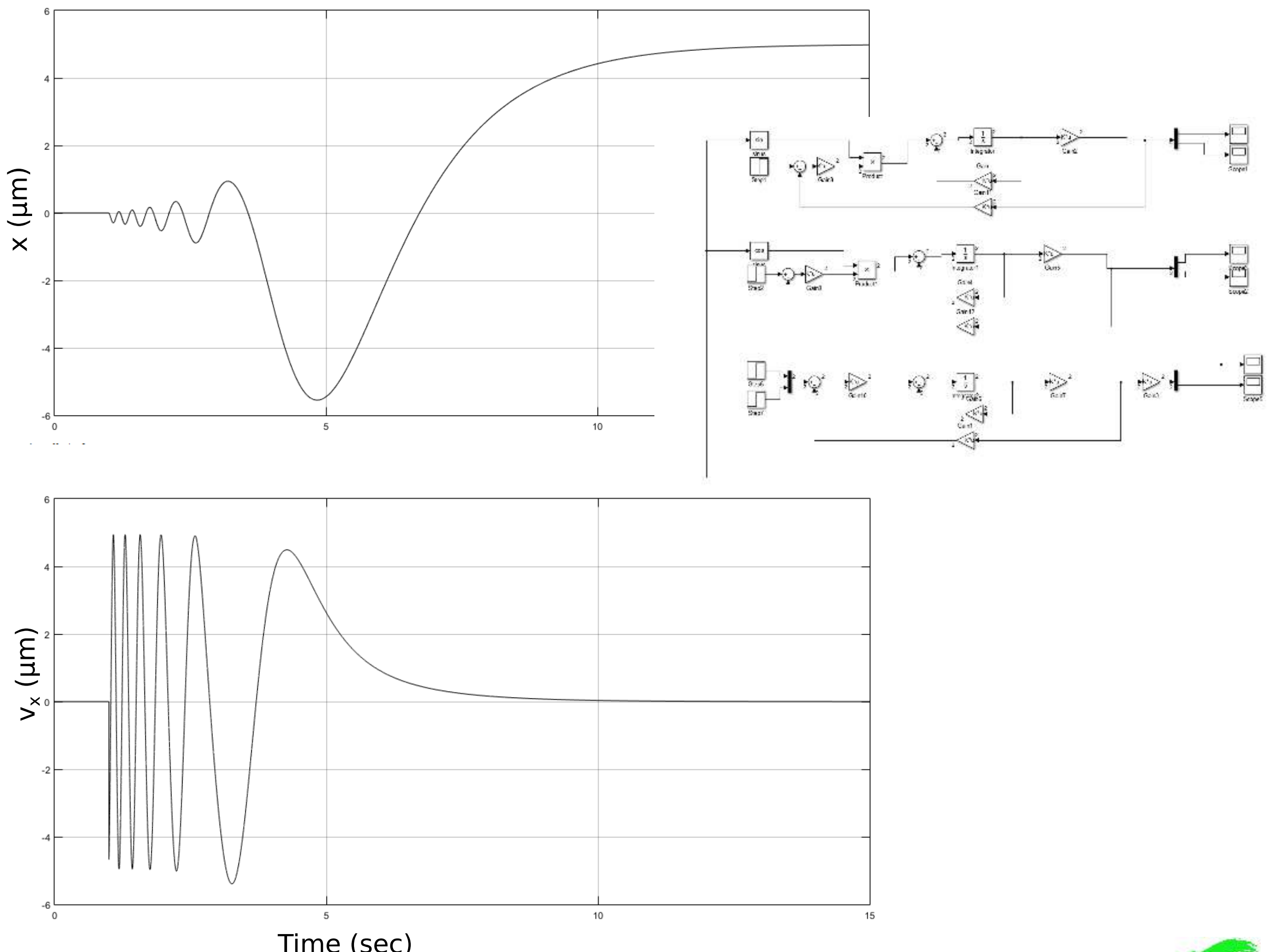
Simu#1: u_0=5µm/s, ||b||=10µT



Simu#2: u_0=5µm/s, ||b||=10mT



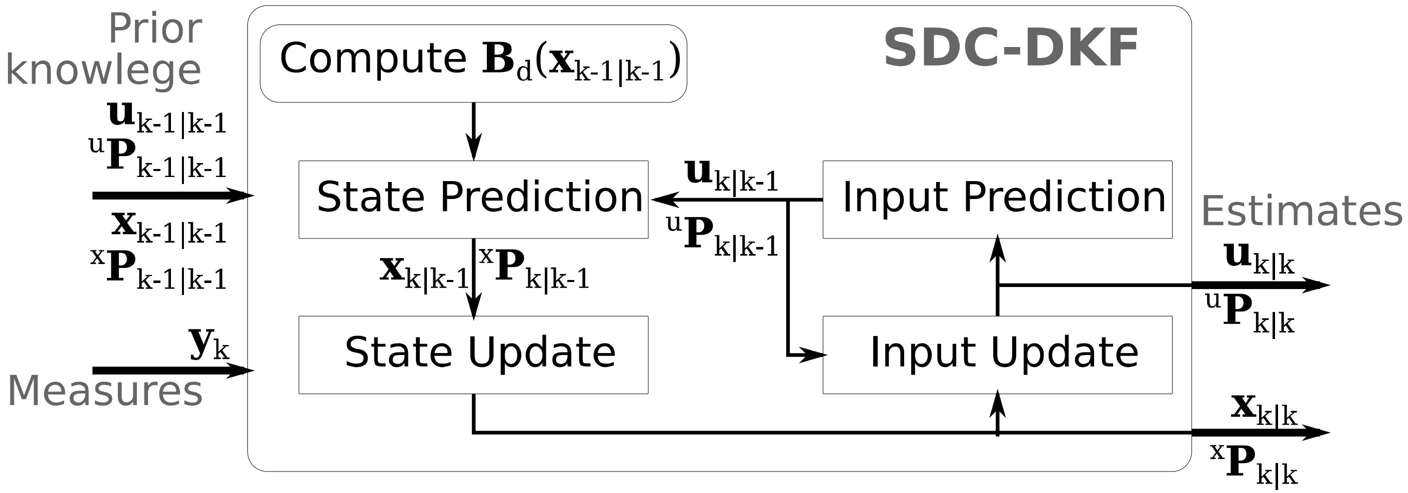
- **Matlab/Simulink simple state feeback control**



❑ Validation: (KF) tracking

$$\begin{cases} \dot{x} = A\,x + B(x)\,u + W \\ \dot{y} = C\,x + v \end{cases}$$

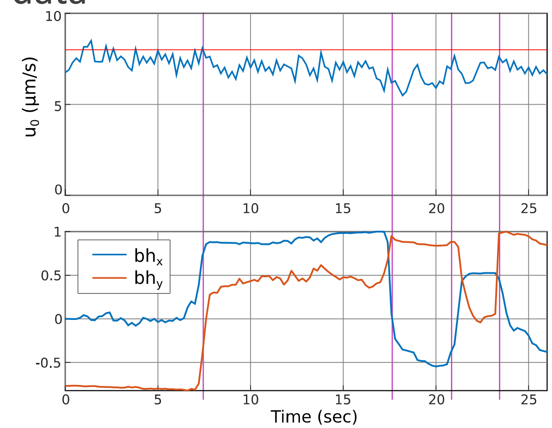
- Kalman Filtering (KF)
  - State-dependent system → SDC-KF
  - Not well-known/ Unknown inputs → Dual-KF

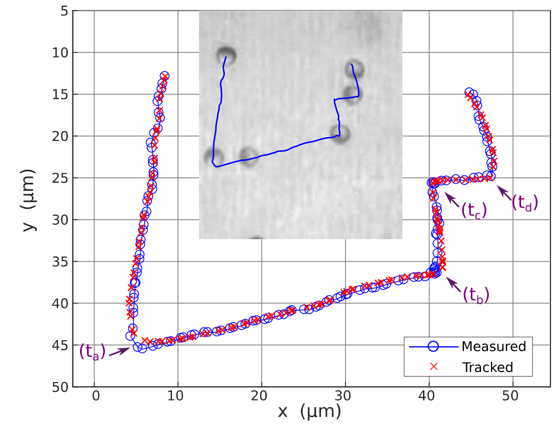


Case study (MJR)

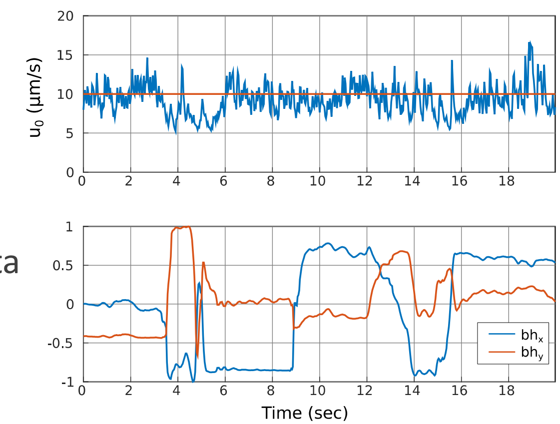
❑ Validation: SDC-KF tracking

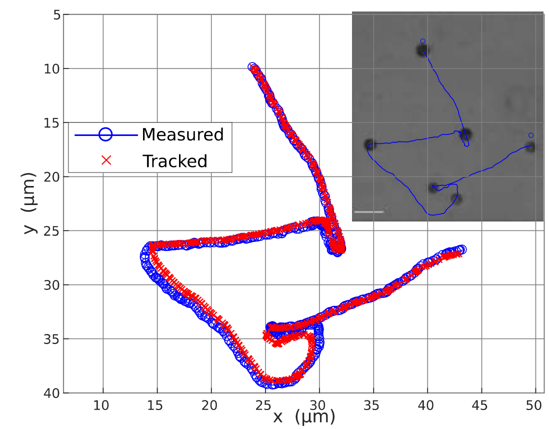
- (Baraban et al. 2012) experimental data
  - Spheroidal catalytic JMR
  - SiO_4+[Co(0.4nm)Pt(0.6nm)]$_5$;
  - radius r≈2.5µm ;
  - u_0≈8µm/s, no magnetic field data





- (Ma et al. 2016) experimental data
  - Urea-powered spherical hollow magnetic microrobot
  - coated with Fe(10nm)Au(3nm) ;
  - radius r≈2.3µm;
  - u_0≈10µm/s, no magnetic field data





❑ Conclusion

- Modeling of self-phoretic particle
- Simulation of MJR dynamic
- SDC-DKF tracking

❑ Prospects

- Characterization/identification of MJRs
- (Advanced) Controller design

Thanks you for your attention

… questions?

Contact and further information:

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